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How does drift wave turbulence convert parallel compression into perpendicular flows?

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Abstract

Zonal flow (ZF) momentum balance in a three-dimensional, coupled drift-ion acoustic wave system is examined. In a three-dimensional system, conservation of potential vorticity (PV) is broken by fluctuating parallel flow compressibility. The coupling between PV fluctuation and fluctuating parallel flow compression defines a source/sink for fluctuating potential enstrophy density, and thus influences the wave momentum density and modifies the zonal momentum theorem. We show that perpendicular ZFs can be excited by stationary turbulence via compressional coupling, even in the absence of a driving force and potential enstrophy flux. The coupling drive involves both perpendicular and parallel dynamics, and does not require symmetry breaking in the turbulence spectrum. A new mechanism for ZF generation is thus revealed.

1. Introduction

It is broadly recognized that zonal flow (ZF) plays a crucial role in regulating turbulence and transport in magnetic confinement plasmas [1, 2]. Over the past two decades, ZF dynamics have been intensively studied, and considerable progress has been achieved. Gyrokinetic simulation studies reported the important role of ZFs in ion temperature gradient (ITG) [3, 4] and collisionless trapped electron mode [5–7] turbulence. Recent experimental results on many devices also identified the crucial role of ZFs in the L–H transition [8–11]. There are also many theoretical works on turbulent flow generation by turbulent Reynolds stress [12, 13]. The flux of potential vorticity (PV) is often used instead of the closely related Reynolds force (divergence of Reynolds stress) to describe ZF excitation. The assumption of symmetry along the zonal coordinate was used to derive the Taylor identity [14], i.e. $\langle v_r \tilde{q} \rangle = \partial \langle v_r v_y \rangle / \partial r$, where $\tilde{q} = \nabla_\perp^2 \phi - \phi$ is the PV fluctuation and $\boldsymbol{v}$ is the normalized fluctuating $\boldsymbol{E} \times \boldsymbol{B}$ velocity. This expression explicitly shows the link between PV transport and Reynolds force. Recently, theoretical studies of ZF dynamics have begun to focus on ZF momentum instead of energy transfer. The focus on momentum leads to non-acceleration theorems [15] for quasi-geostrophic and drift wave turbulence in real space [16], and for gyrokinetic drift wave turbulence in phase space [17]. The theorems provide an approach to understand ZF dynamics in terms of flux drive and turbulence spreading rather than spectral transfer.

Most existing theoretical studies of ZF focused only on perpendicular dynamics, but neglected the dynamical coupling along the magnetic field line. Usually, $k_\parallel L_n \sim O(\epsilon)$ justifies the neglect of the dynamical parallel coupling. Here, $k_\parallel$ is the wavenumber along the magnetic field line, L_n is the density gradient scale length and ϵ is the standard ordering parameters for low frequent drift wave turbulence. Two popular drift wave models, i.e. the Hasegawa–Mima (H–M) model [18] and the Hasegawa–Wakatani (H–W) model [19] are often used to investigate drift wave–ZF system, in which PV is conserved up to diffusion. PV obeys a two-dimensional (2D) equation in these two models, although the H–W system itself is explicitly three-dimensional (3D). The constraint of PV conservation on ZF evolution in H–W system relates the sum of the negative of the pseudomomentum and the ZF momentum to the particle flux, the potential enstrophy dissipation, the potential enstrophy density flux and the drag of ZF. In this case, stationary turbulence cannot excite a ZF in the absence of particle flux, dissipation and transport of potential enstrophy [16].

However, the drift waves will couple to the ion acoustic waves if the parallel wave number $k_\parallel$ is big enough. In such a case, the coupling between perpendicular and parallel dynamics could result in a qualitative difference from the 2D system. In a 3D coupled drift-ion acoustic wave system, PV is no longer conserved due to the compression of fluctuation parallel velocity. One could then ask how the zonal momentum theorem obtained in [16] will be modified, once parallel dynamics is included? Therefore, it is interesting and important to study the effects of the coupling between drift waves and ion acoustic waves on ZF dynamics.

In this work, we study ZF dynamics in a 3D coupled drift-ion acoustic wave system. In the evolution equation for fluctuating potential enstrophy density, the coupling between drift waves and ion acoustic waves acts as a source/sink. As a consequence, the coupling affects the wave momentum density (turbulence pseudomomentum). By combining the evolution equation of the wave momentum density and the mesoscale ZF equation, we obtain a zonal momentum theorem—an extension of the Charney–Drazin theorem—for this 3D system. The difference from the 2D equation is that stationary turbulence can excite ZFs via the coupling of drift waves and ion acoustic waves even in the absence of a driving force and a potential enstrophy flux. This means that the drift-ion acoustic coupling can effectively convert parallel compression into perpendicular flow. This coupling drive does not require symmetry breaking in the turbulence spectrum. Thus, we find a new mechanism for ZF generation, which is different from turbulent stress generation. We will discuss the roles of different types of symmetry breaking in flow generation and the relationship between perpendicular and parallel dynamics in the conclusion.

The rest of this paper is organized as follows. In section 2, we present the derivation of the zonal momentum theorem for the 3D coupled drift-ion acoustic wave system, and evaluate the coupling driving source of the ZF in shearless slab geometry and sheared slab geometry, respectively. In section 3, we summarize our work and give some discussions. In the appendix, we present a derivation of the wave momentum density for shearless slab geometry.

2. ZF momentum in 3D coupled drift-ion acoustic wave system

In this work, we adopt the nonlinear equations which describe coupled drift waves and ion acoustic waves in 3D [20],

$$\frac{\partial}{\partial t}\left(\nabla_\perp^2\phi-\phi\right)+\hat{z}\times\nabla\phi\cdot\nabla\nabla_\perp^2\phi+\left(\frac{\partial}{\partial r}\ln n_0\right)\frac{\partial}{\partial y}\phi$$
$$=\tilde{f}+\mu\nabla^2\left(\nabla_\perp^2\phi-\phi\right)+\nabla_\parallel u_\parallel, \quad (1)$$

$$\left(\frac{\partial}{\partial t}+\hat{z}\times\nabla\phi\cdot\nabla\right)u_\parallel=-\nabla_\parallel\phi. \quad (2)$$

Here we have used the standard normalization for electric potential fluctuation $\phi\equiv e\tilde{\phi}/T_e$, parallel velocity fluctuation $u_\parallel\equiv\tilde{u}_\parallel/c_s$, spatial scales $\nabla_\perp\equiv\rho_s\nabla_\perp$ and $\nabla_\parallel\equiv\rho_s\nabla_\parallel$, time scale $t\equiv\omega_{ci}t$ with $\omega_{ci}=eB/(m_ic)$ is the ion gyrofrequency, c_s is the ion acoustic velocity and ρ_s is the ion Larmour radius at the electron temperature. A standard ordering for low-frequency drift wave turbulence is given by

$$k_\perp\rho_s\sim 1, \quad (3)$$

where $k_\perp$ is the perpendicular wave number, and

$$\frac{\omega}{\omega_{ci}}\sim\phi\sim\frac{\rho_s}{L_n}\sim\epsilon. \quad (4)$$

But, a different parallel ordering $k_\parallel^2L_n^2\sim O(\epsilon)$ rather than $k_\parallel L_n\sim O(\epsilon)$ is used in this work to highlight the role of parallel dynamics. Equations (1) and (2) assume cold ion, i.e. $T_e\gg T_i$. The first two terms on the rhs of equation (1) correspond to forcing and dissipation, respectively. The last term on the rhs of equation (1) comes from the compression of fluctuating ion parallel flow. This formally smaller term breaks the conservation of PV, which will lead to a qualitatively different result. In the parallel velocity equation, i.e. equation (2), we assumed isothermal electrons. ∇P_i is absent due to the cold ion approximation.

We use the notation $\tilde{q}=\nabla_\perp^2\phi-\phi$ for fluctuating PV. Multiplying equation (1) by $\tilde{q}$ and then taking a zonal average, we obtain the fluctuating potential enstrophy balance equation

$$\frac{\partial}{\partial t}\left\langle\frac{\tilde{q}^2}{2}\right\rangle+\frac{\partial}{\partial r}\left\langle v_r\frac{\tilde{q}^2}{2}\right\rangle$$
$$=\left(\frac{\partial}{\partial r}\ln n_0\right)\langle v_r\tilde{q}\rangle+\langle\tilde{f}\tilde{q}\rangle-\mu\langle(\nabla\tilde{q})^2\rangle+\left\langle\tilde{q}\nabla_\parallel u_\parallel\right\rangle. \quad (5)$$

Here, v_r is the fluctuating radial $\boldsymbol{E}\times\boldsymbol{B}$ velocity, and the dissipation of zonal averaged potential enstrophy $\mu\partial^2\langle\tilde{q}^2/2\rangle/\partial r^2$ is neglected. This is because the scale of zonal averaged quantity is mesocale which is lager than the corresponding drift wave scale. As discussed in [16], the potential enstrophy is produced by forcing and by PV flux acting on density gradient, damped by viscosity and transported by advection. However, we note that, there is an additional term, the coupling between PV and compressibility of parallel flow fluctuation, which can act as a source or sink for potential enstrophy. This coupling changes the potential enstrophy balance equation obtained in 2D system. It is therefore natural to examine its effect on the Charney–Drazin theorem.

The zonally averaged flow evolves according to

$$\frac{\partial}{\partial t}\langle v_y\rangle=-\left\langle v_r\nabla_\perp^2\phi\right\rangle-\nu\langle v_y\rangle. \quad (6)$$

Here, the Taylor identity [14] $\partial\langle v_rv_y\rangle/\partial r=\langle v_r\nabla_\perp^2\phi\rangle$ is used.

Combining equations (5) and (6), a zonal momentum theorem for 3D coupled drift-ion acoustic wave system is easily obtained

$$\frac{\partial}{\partial t}\left[\langle v_y\rangle+\left(\frac{\partial\ln n_0}{\partial r}\right)^{-1}\left\langle\frac{\tilde{q}^2}{2}\right\rangle\right]=-\nu\langle v_y\rangle+\left(\frac{\partial\ln n_0}{\partial r}\right)^{-1}$$
$$\times\left[-\frac{\partial}{\partial r}\left\langle v_r\frac{\tilde{q}^2}{2}\right\rangle+\langle\tilde{f}\tilde{q}\rangle-\mu\langle(\nabla\tilde{q})^2\rangle+\left\langle\tilde{q}\nabla_\parallel u_\parallel\right\rangle\right]. \quad (7)$$

Here, the PV flux in equation (5) and vorticity flux in equation (6) cancel each other, since $\langle v_r\phi\rangle=0$. Similar to 2D drift wave system, the second term on the lhs of equation (7) is also recognized as the negative of wave momentum density (negative of the turbulence pseudomomentum), i.e.

$$\left(\frac{\partial \ln n_0}{\partial r}\right)^{-1}\left\langle\frac{\tilde{q}^2}{2}\right\rangle = -\frac{1}{m_{\mathrm{i}}c_{\mathrm{s}}}\sum_k k_y\frac{E_k}{\omega_k}. \tag{8}$$

But the difference is both the wave energy density E_k and the linear frequency ω_k include the parallel ion acoustic correction. We will show this in the appendix.

Equation (7) generalizes the classical Charney–Drazin non-acceleration theorem [15, 16] from 2D system to 3D coupled drift-ion acoustic wave system. It shows explicit link of ZF evolution to the wave momentum density evolution, the forcing drive the transport and dissipation of potential enstrophy, the drag and the coupling of drift waves to ion acoustic waves. All the elements in the 2D momentum theorem are included in this generalized momentum theorem, but the last term on the rhs of equation (7), i.e. the coupling term, is a new one. This new term provides a driving source for ZF momentum. For stationary states, the ZF can be written as

$$\langle v_y\rangle = \frac{1}{\nu}\left(\frac{\partial \ln n_0}{\partial r}\right)^{-1} \times\left[-\frac{\partial}{\partial r}\left\langle v_r\frac{\tilde{q}^2}{2}\right\rangle + \langle\tilde{f}\tilde{q}\rangle - \mu\langle(\nabla\tilde{q})^2\rangle + \langle\tilde{q}\nabla_\parallel u_\parallel\rangle\right]. \tag{9}$$

This expression shows that stationary turbulence can excite ZF via the coupling drive, even in the absence of force driving, potential enstrophy flux and dissipation. The coupling term $\langle\tilde{q}\nabla_\parallel u_\parallel\rangle$ is negative definite, i.e. its product with $(\partial \ln n_0/\partial r)^{-1}$ is positive definite for peaked density profile. This will be shown in the following two subsections. Therefore, the saturated ZF level increases in comparison with the 2D system because of the coupling drive. We show that the coupling between drift waves and ion acoustic waves plays an important role of converting parallel compression into perpendicular flow. This is a new mechanism for ZF generation. Now, we estimate the coupling drive in two kinds of geometries.

2.1. Shearless slab geometry

First, we consider the simplest case, i.e. shearless slab geometry. By linearizing equations (1) and (2), we obtain

$$\mathrm{i}\omega_k\left(1+k_\perp^2\rho_{\mathrm{s}}^2\right)\phi_k - \mathrm{i}\omega_{*n}\phi_k - \mathrm{i}k_\parallel c_{\mathrm{s}}u_k = 0, \tag{10}$$

$$-\mathrm{i}\omega_k u_k + \mathrm{i}k_\parallel c_{\mathrm{s}}\phi_k = 0, \tag{11}$$

where, $\omega_{*n} = k_y\rho_{\mathrm{s}}c_{\mathrm{s}}/L_n$. The dispersion relation is as follows:

$$\left[\left(1+k_\perp^2\rho_{\mathrm{s}}^2\right) - \frac{\omega_{*n}}{\omega_k} - \frac{k_\parallel^2 c_{\mathrm{s}}^2}{\omega_k^2}\right]\phi_k = 0. \tag{12}$$

In this equation, the first two terms are dominant, from which we can obtain the familiar electron drift wave frequency $\omega_* = \omega_{*n}/(1+k_\perp^2\rho_{\mathrm{s}}^2)$. The last term is the correction from ion acoustic waves. So we write $\omega_k = \omega_* + \delta\omega_k$, with $|\delta\omega_k| \ll \omega_*$, and to first order in small quantities find:

$$\frac{\omega_{*n}\delta\omega_k}{\omega_*^2} - \frac{k_\parallel^2 c_{\mathrm{s}}^2}{\omega_*^2} = 0. \tag{13}$$

The ordering here is consistent with what we mentioned before, i.e. $\delta\omega_k/\omega_* \sim k_\parallel^2 L_n^2 \sim \epsilon$. Then, we obtain the drift wave frequency with the ion acoustic correction

$$\omega_k = \frac{\omega_{*n}}{\left(1+k_\perp^2\rho_{\mathrm{s}}^2\right)} + \frac{k_\parallel^2 c_{\mathrm{s}}^2}{\omega_{*n}}. \tag{14}$$

Now, we calculate the coupling between drift waves and ion acoustic waves in the zonal momentum theorem, i.e. the last term in equation (7):

$$\begin{aligned}\langle\tilde{q}\nabla_\parallel u_\parallel\rangle &= -\Re\sum_k\left(1+k_\perp^2\rho_{\mathrm{s}}^2\right)\frac{\mathrm{i}}{\omega_k+\mathrm{i}\,|\triangle\omega_k|}k_\parallel^2\rho_{\mathrm{s}}c_{\mathrm{s}}\,|\phi_k|^2\\ &= -\sum_k\frac{|\triangle\omega_k|}{\omega_k^2}\left(1+k_\perp^2\rho_{\mathrm{s}}^2\right)k_\parallel^2\rho_{\mathrm{s}}c_{\mathrm{s}}\,|\phi_k|^2.\end{aligned} \tag{15}$$

Here, we consider finite amplitude stationary turbulence and thus, a finite correlation time. $|\triangle\omega_k|$ is decorrelation rate due to $\boldsymbol{E}\times\boldsymbol{B}$ nonlinearity of the parallel flow in equation (2). The wave absorption from $\omega - k_yV_E(r)$ resonance is neglected. It is obvious that the coupling does not vanish and no symmetry breaking in the turbulence spectrum is required to render $\langle\tilde{q}\nabla_\parallel u_\parallel\rangle \neq 0$. Therefore, the coupling can convert parallel compressibility into ZF. This is a new mechanism for ZF generation, which is *qualitatively* different from the 2D Reynolds forcing drive.

To semi-quantitatively illustrate the important role of parallel dynamics, we compare the coupling term, $\langle\tilde{q}\nabla_\parallel u_\parallel\rangle$, with the PV flux term, $(\frac{\partial}{\partial r}\ln n_0)\langle v_r\tilde{q}\rangle$. The former one is given on the rhs of equation (15). The latter one is the usual Reynolds forcing drive for ZF in 2D system, which can be estimated as

$$\left(\frac{\partial}{\partial r}\ln n_0\right)\langle v_r\tilde{q}\rangle \sim \frac{\rho_{\mathrm{s}}}{L_n}\frac{q^2c_{\mathrm{s}}^2}{\gamma_k\omega_{\mathrm{ci}}}k_y^2\rho_{\mathrm{s}}^2|\phi_k|^2\frac{V_{\mathrm{ZF}}}{c_{\mathrm{s}}}, \tag{16}$$

by the modulation calculation [1]. Here, q is the radial wave number of ZF, which is mesoscale, can be typically estimated as $q^{-1} \sim \sqrt{\rho_{\mathrm{s}}L_n}$, γ_k is the linear growth of the ambient turbulence, which can be estimated as $|\triangle\omega_k|$, i.e. $\gamma_k/\omega_{\mathrm{ci}} \sim |\triangle\omega_k|/\omega_{\mathrm{ci}} \sim (q\rho_{\mathrm{s}})^{1/2}(\rho_{\mathrm{s}}/L_n)$, the ZF velocity can be estimated as $V_{\mathrm{ZF}}/c_{\mathrm{s}} \sim q\rho_{\mathrm{s}}^2/L_n$. Then, the ratio between coupling term and PV flux term is

$$\frac{\langle\tilde{q}\nabla_\parallel u_\parallel\rangle}{\left(\frac{\partial}{\partial r}\ln n_0\right)\langle v_r\tilde{q}\rangle} \sim k_\parallel^2 L_n^2\epsilon^{-1}, \tag{17}$$

where $k_\perp\rho_{\mathrm{s}} \sim 1$ was used. Now, we can see that $k_\parallel^2 L_n^2 \sim \epsilon$ makes the coupling term as the same order as the PV flux term. It means that these two terms are comparable for the ordering adopted in this work. Therefore, the parallel coupling term is qualitatively different from, but quantitatively comparable to, the 2D Reynolds forcing drive for ZF.

2.2. Sheared slab geometry

In this subsection, we consider sheared slab geometry. We use $k_\parallel = k_yx/L_s$, with $x = r - r_0$, where r_0 is the radial location of the resonant surface and L_s is the magnetic shear scale length. Linearizing equations (1) and (2) again, we have

$$\mathrm{i}\omega_k\left(1+k_y^2\rho_s^2-\rho_s^2\frac{\partial^2}{\partial x^2}\right)\phi_k-\mathrm{i}\omega_{*n}\phi_k-\mathrm{i}k_yc_s\frac{x}{L_s}u_k=0, \tag{18}$$

$$-\mathrm{i}\omega_ku_k+\mathrm{i}k_yc_s\frac{x}{L_s}\phi_k=0. \tag{19}$$

Substitution of equation (19) into equation (18) yields the linear eigenmode equation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{1}{\rho_s^2}\left(\frac{\omega_{*n}}{\omega_k}-1-k_y^2\rho_s^2\right)+\frac{k_y^2\omega_{ci}^2}{L_s^2\omega_k^2}x^2\right]\phi_k=0. \tag{20}$$

This is a Weber equation, and the solution is Hermite function

$$\phi_k(x)=\phi_k\exp\left[-\frac{\sigma x^2}{2}\right]H_l\left[\sqrt{\sigma}x)\right], \tag{21}$$

where, $\sigma=\pm\mathrm{i}k_y\omega_{ci}/(L_s\omega_k)$. If we take $l=0$, $H_0=1$, the solution becomes

$$\phi_k(x)=\phi_k\exp\left[\mp\mathrm{i}\frac{k_y\omega_{ci}}{2L_s\omega_k}x^2\right], \tag{22}$$

so that

$$k_x=-\mathrm{i}\frac{\partial}{\partial x}=\mp\frac{k_y\omega_{ci}}{L_s\omega_k}x, \tag{23}$$

and thus

$$\omega_k=\mp\frac{k_y\omega_{ci}}{L_sk_x}x. \tag{24}$$

Therefore, the group velocity in the x direction is

$$v_{gx}=\frac{\partial\omega_k}{\partial k_x}=\pm\frac{k_y\omega_{ci}}{L_sk_x^2}x. \tag{25}$$

Outgoing wave propagation requires $v_{gx}/x>0$, i.e. we should take the upper and lower solutions for $k_y>0$ and $k_y<0$, respectively. Finally, the solution of the eigenmode equation is

$$\phi_k(x)=\phi_k\exp\left[-\mathrm{i}\frac{|k_y|c_s}{2L_s\omega_k\rho_s}x^2\right]. \tag{26}$$

Now, the coupling between drift waves and ion acoustic waves is estimated using the quasilinear approximation in the following:

$$\begin{aligned}\langle\tilde{q}\nabla_\parallel u_\parallel\rangle&=-\Re\sum_k\left[1+k_y^2\rho_s^2+\frac{k_y^2c_s^2}{L_s^2\omega_k^2}x^2-\mathrm{i}\frac{|k_y|\rho_sc_s}{L_s\omega_k}\right]\\&\quad\times\frac{\mathrm{i}}{(\omega_k+\mathrm{i}\,|\triangle\omega_k|)}k_y^2\rho_sc_s\frac{x^2}{L_s^2}\,|\phi_k|^2=-\sum_k k_y^2\rho_sc_s\frac{\triangle^2}{L_s^2}\\&\quad\times\left\{\left[\left(1+k_y^2\rho_s^2\right)+\frac{k_y^2c_s^2}{\omega_k^2}\frac{\triangle^2}{L_s^2}\right]\frac{|\triangle\omega_k|}{\omega_k^2}+\frac{\rho_s}{L_s}\frac{|k_y|c_s}{\omega_k^2}\right\}|\phi_k|^2.\end{aligned} \tag{27}$$

Here, $\triangle$ is the radial width of the potential fluctuation spectrum $|\phi_k|^2=F(x/\triangle)$, i.e. $\triangle^2=\int\mathrm{d}x x^2F/\int\mathrm{d}xF$, with $x=r-r_0$. By using $k_\parallel L_n\sim k_y\triangle L_n/L_s\sim\epsilon^{1/2}$, we can obtain $\triangle/L_s\sim\epsilon^{3/2}$. Similar to the shearless slab case, non-zero coupling does not require any symmetry breaking in the turbulence spectrum. It can also convert parallel compression into ZF. The magnitude of ZF drive due to this coupling is dependent on the spectral width and hence on the magnetic shear scale length.

3. Conclusion and discussion

In this work, we investigated the zonal momentum theorem in a 3D coupled drift-ion acoustic wave system. This system is different from the familiar Hasegawa–Mima system in 2D, for which PV is conserved up to diffusion. In the 3D system, the compression of fluctuating parallel flow breaks the conservation of PV. Consequently, the coupling between drift waves and ion acoustic waves acts as a potential enstrophy density source or sink, and thus influences the wave momentum density (the turbulence pseudomomentum). We derive a zonal momentum theorem by combining the equation of wave momentum density and the ZF equation for a 3D system. We take into account the parallel dynamics which was neglected in most of the previous works on this subject.

We find that, in the zonal momentum theorem for the 3D system, the coupling between drift waves and ion acoustic waves which is absent in 2D acts as a driving source. Thus, it is possible for stationary turbulence to excite ZF by this coupling even in the absence of force driving and potential enstrophy flux. Therefore, in this sense, the non-acceleration theorem for ZF momentum obtained from the 2D equation [15] is modified in this work. Via the acoustic coupling, drift waves can convert parallel compression into perpendicular flow.

We evaluate the driving term using the quasilinear approximation for the simplest cases of shearless slab geometry and for sheared slab geometry, respectively. For both cases, the non-zero coupled driving does not require special symmetry breaking in the turbulence spectrum. This is in contrast to the poloidal Reynolds stress or toroidal Reynolds stress driven flow, and follows from the fact that conservation of PV is explicitly broken in the 3D nonlinear coupled equations. If we think of the symmetry breaking needed in poloidal (toroidal) Reynolds stress drive as a case of 'spontaneous symmetry breaking', the breaking of conservation of PV might then be considered as 'dynamical symmetry breaking'. In this sense, we uncover a new mechanism for ZF generation which is different from purely 2D turbulent stress drive.

One may naturally relate this work to parallel flows and intrinsic rotation, since the $k_\parallel$ symmetry breaking induced by $\boldsymbol{E}\times\boldsymbol{B}$ shear can accelerate a net parallel flow, as presented in [21–23]. In our work, we find that ZF can be generated by coupling of drift waves and ion acoustic waves, even with a symmetric turbulence spectrum. Then ZF shear will lead to intrinsic rotation, according to [21–23]. It forms a distinct, dynamical pathway which is shown in figure 1. We also note that the coupling drive of ZF is independent of the sign of the fluctuation frequency, which can be seen from equations (15) and (27). Therefore, a similar conclusion can also be obtained by applying this theory to ion temperature gradient (ITG)–ion acoustic coupling system. This suggests that the intrinsic rotation might be universal, if dynamical parallel coupling is included.

One may also interested in the effects of parallel flow shear on ZF dynamics. Self-consistent generation of sheared perpendicular flow in parallel shear flow driven turbulence has been studied in [24]. Here, we discuss the effects of weak parallel flow shear, i.e. $k_y\rho_s(\partial U_0/\partial r)\sim k_\parallel c_s$ on zonal

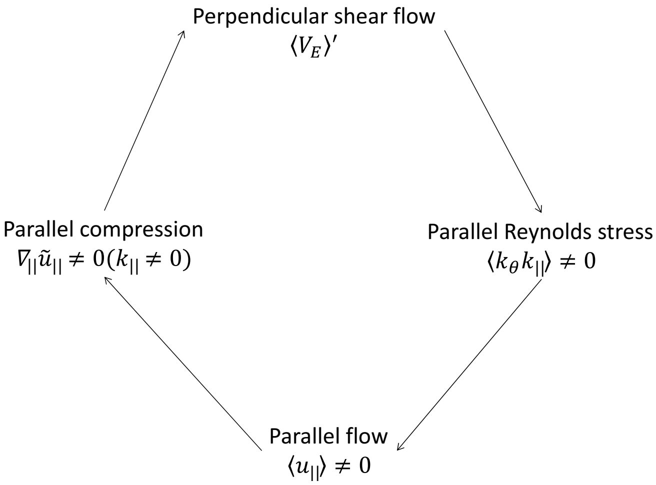

Figure 1. The pathway from parallel compressional coupling to parallel flow.

momentum balance. One consequence is the second term on the lhs of equation (7) can be reduced to a negative value of wave momentum density only to the lowest order. Unlike the limit discussed in the Appendix, the corrections from weak parallel shear to the wave momentum energy and the wave frequency do not cancel each other exactly. By a simple manipulation, the normalized wave momentum density can be written as

$$\frac{1}{m_i c_s}\sum_k k_y N_k = -\left(\frac{\partial \ln n_0}{\partial r}\right)^{-1}\frac{\langle\tilde{q}^2\rangle}{2}\left(1 - O(k_\parallel^2 L_n^2)\right). \quad (28)$$

We can see the additional term in wave momentum density introduced by weak parallel flow shear is smaller by one order. The other consequence appears in the coupling drive $\tilde{q}\nabla_\parallel u_\parallel$. For the case of sheared slab geometry, in equation (27), the weak parallel flow shear will introduce an additional term

$$\sum_k k_y^2 \rho_s c_s \frac{\rho_s}{c_s}\frac{\partial U_0}{\partial r}\left\{\left[\left(1 + k_y^2\rho_s^2\right)\frac{x}{L_s} + \frac{k_y^2 c_s^2}{\omega_k^2}\frac{x^3}{L_s^3}\right] \times \frac{|\triangle\omega_k|}{\omega_k^2} + \frac{x}{L_s}\frac{\rho_s}{L_s}\frac{|k_y|c_s}{\omega_k^2}\right\}|\phi_k|^2.$$

We note that this term vanishes for the case of a symmetric turbulence spectrum. This is in contrast to the original coupling drive term which does not require asymmetric turbulence. Proceeding as in the study of intrinsic torque caused by turbulence intensity gradient, we take $I(x) = |\phi_k|^2(x) = I(0) + x(\partial I/\partial r)$ [25], so the additional term becomes

$$\frac{L_s}{L_I}\frac{\rho_s}{c_s}\frac{\partial U_0}{\partial r}\sum_k k_y^2\rho_s c_s \frac{\triangle^2}{L_s^2}\left\{\left[\left(1 + k_y^2\rho_s^2\right) + \frac{k_y^2 c_s^2}{\omega_k^2}\frac{\triangle^2}{L_s^2}\right] \times \frac{|\triangle\omega_k|}{\omega_k^2} + \frac{\rho_s}{L_s}\frac{|k_y|c_s}{\omega_k^2}\right\}|\phi_k|^2.$$

Here, $L_I^{-1} = \partial \ln I/\partial r$ is the intensity gradient length scale. Comparing with the original term on the rhs of equation (27), this additional term is order of $(L_s/L_I)(\rho_s\partial U_0)/(c_s\partial r)$. Using weak parallel flow shear assumption $k_y\rho_s(\partial U_0/\partial r) \sim k_\parallel c_s$ mentioned above, we obtain $(L_s/L_I)(\rho_s\partial U_0)/(c_s\partial r) \sim \triangle/L_I \sim \epsilon^{3/2}(L_s/L_I)$. For the case with normal magnetic shear, L_s and L_I are roughly the major radius R_0 and density gradient scale length L_n, respectively. L_n is roughly as the same order as the minor radius except for a steep barrier regime, i.e. $L_s/L_I \lesssim \epsilon^{-1/2}$, the order is thus reduced to $\triangle/L_I \lesssim \epsilon$. It means that the additional term is smaller than the original term. Hence, the changes due to weak parallel flow shear do not substantively affect the zonal momentum balance.

In conclusion, the Charney–Drazin non-acceleration theorem has been generalized from 2D systems to 3D coupled drift-ion acoustic wave systems. The parallel compressional coupling which breaks PV conservation can excite perpendicular ZF, even for stationary symmetric turbulence.

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Appendix A. Derivation of wave momentum density in shearless slab geometry

The wave energy density in the 3D coupled drift-ion acoustic wave system can be written as

$$E_k = \frac{T_e}{2}\left[\left(1 + k_\perp^2\rho_s^2\right)|\phi_k|^2 + \left|u_k^2\right|\right]. \quad (A.1)$$

Here, the last term comes from the ion acoustic waves. By linearization of equation (2), we obtain

$$u_k = \frac{k_\parallel c_s}{\omega_k}\phi_k. \quad (A.2)$$

Thus, the wave energy density becomes

$$E_k = \frac{T_e}{2}\left(1 + k_\perp^2\rho_s^2 + \frac{k_\parallel^2 c_s^2}{\omega_k^2}\right)|\phi_k|^2. \quad (A.3)$$

The wave action density is defined as

$$N_k = \frac{E_k}{\omega_k}. \quad (A.4)$$

Here, ω_k is the wave frequency for shearless slab geometry is given by equation (14) in the section 2.1. So the wave action density can be obtained by substituting equations (14) and (A.3) into the preceding equation

$$N_k = \frac{T_e}{2}\frac{\left(1 + k_\perp^2\rho_s^2\right)^2}{\omega_{*n}}|\phi_k|^2. \quad (A.5)$$

Note that the lowest order correction from ion acoustic effects to the wave action density vanishes, because ion acoustic corrections to wave energy density and the linear wave frequency cancel each other.Then, the normalized wave momentum density or turbulence pseudomomentum can be written as

$$\frac{1}{m_{\mathrm{i}}c_{\mathrm{s}}}\sum_k k_y N_k = \sum_k \frac{\left(1+k_\perp^2\rho_{\mathrm{s}}^2\right)^2}{2\rho_{\mathrm{s}}/L_n}\,|\phi_k|^2 = -\left(\frac{\partial \ln n_0}{\partial r}\right)^{-1}\frac{\langle \tilde{q}^2\rangle}{2}. \tag{A.6}$$

This is the second term on the lhs of equation (7), with opposite sign.

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